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Contribution to the Sensitivity Coefficients Analysis in the Extended Dynamic Plane Source (EDPS) Method

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This work reports on a method for measuring thermophysical properties (thermal conductivity and diffusivity) of materials. The theory of the dynamic plane source method and experimental apparatus is described. The contribution of this work is the determination of the time interval within which the fitting procedure should be applied. A new algorithm for sensitivity coefficient analysis is presented, and the results are compared with those of a difference analysis of experiment modelling.

KEY WORDS: difference analysis; dynamic plane source method; sensitivity coefficient; thermal conductivity; thermal diffusivity; thermophysical properties.

1. INTRODUCTION

Development of new materials and advancement of materials engineering have influenced the development of measurement methods of physical properties over the last several decades. Thermophysical properties are some of the most important material properties. Progress of electronics and computer technologies has resulted in a transition from stationary to unstationary methods. Transient methods [1] are based on generation of a dynamic temperature field inside the specimen. The measuring process can be described as follows. The temperature of the specimen is stabilized and uniform. Then the dynamic heat flow in the form of a pulse or step-wise function is generated inside the specimen. From the temperature response to this small disturbance, the thermophysical parameters of the specimen can be determined.

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2. EXPERIMENTAL

The extended dynamic plane source (EDPS) method is arranged for one-dimensional heat flow into a finite sample. The configuration of the experiment is obvious from Fig.1. The plane source (PS) disc, which simultaneously serves as the heat source and thermometer, is made of a nickel film covered from both sides with a kapton layer. The heat in the form of a step-wise function is produced by the passage of an electrical current through the PS disc. Two identical samples of cylindrical shape cause symmetrical division of the heat flow into a very good heat conducting material (heat sink), which provides isothermal boundary conditions of the experiment. This method appears to be useful for simultaneous determination of thermal diffusivity a and thermal conductivity λ of low thermally conducting materials.

Figure 2 shows the theoretical temperature function which is a solution of the partial differential equation with boundary and initial conditions corresponding to the experimental arrangement. The temperature function is given by [2]

$$T\,(t) = \frac{ql}{\lambda\sqrt{\pi}} F\,(\Theta, t)\,, \tag{1}$$

where

$$F\,(\Theta, t) = \sqrt{\frac{t}{\Theta}} \left(1 + 2\sqrt{\pi} \sum_{n=1}^{\infty} \beta^n \mathrm{ierfc} \left(n \sqrt{\frac{\Theta}{t}} \right) \right). \tag{2}$$

q is the heat current density, λ is the thermal conductivity and Θ is the characteristic time of the sample given by

$$\Theta = l^2/a, \tag{3}$$

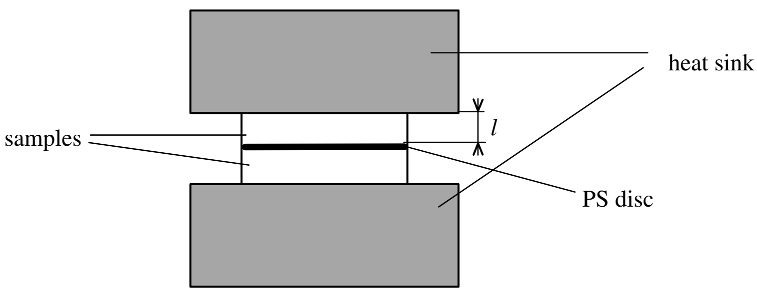

Fig. 1. Setup of the experiment.

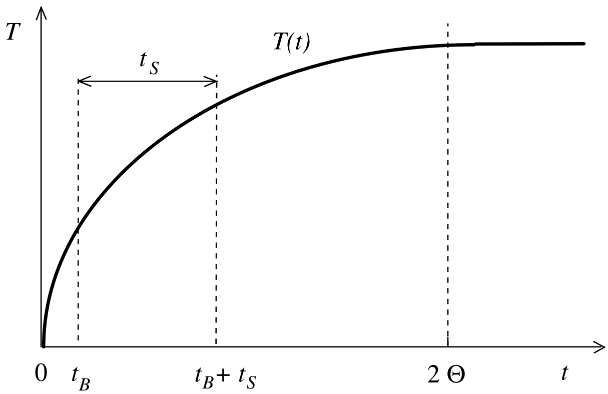

Fig. 2. Temperature function–temperature increase as a function of time.

where l is the thickness and a is the thermal diffusivity of the specimen. Parameter β describes the heat sink imperfection, and ierfc is the error function integral [3].

The principle of the method is based on fitting the theoretical temperature function over the experimental points. The fitting procedure is based on a linear regression [2,4]. The plot of experimental points T_i versus $F(\Theta, t_i)$, calculated using Eq. (2), should be a straight line if Θ has its proper value. Equation (1) predicts a zero intercept but real measurements showed a nonzero offset temperature value τ, defined as the additional increase in the temperature of the PS disc due to its imperfections. The proper value of Θ can be found by using an iterative procedure such that we will change the characteristic time Θ until the correlation coefficient calculated from T_i and $F(\Theta, t_i)$ reaches its maximum. The slope of this straight line gives λ while the iterated Θ gives a.

3. SENSITIVITY COEFFICIENTS ANALYSIS

The sensitivity coefficient is a measure of the change in temperature due to the variation of the estimated parameters. The sensitivity coefficient β_p is defined by [4]

$$\beta_p = p \frac{\partial T(t)}{\partial p}, \tag{4}$$

where p is the parameter to be analyzed and $T(t)$ is the temperature function. The fitting procedure does not work properly when sensitivity coefficients are small or linearly dependent on each other. Therefore, an analysis of the sensitivity coefficients determines the time window in which the evaluation technique can be applied to the temperature response.

In this section we will concentrate on investigating the linear dependence of the sensitivity coefficients. As mentioned in the previous section, there are three parameters whose values should be estimated. They are two thermophysical parameters of the material, λ and a, and the baseline of temperature function τ. Hence, the temperature function in Eq. (1) can be expressed as

$$T(t,a,\lambda,\tau)=\frac{q}{\lambda}\sqrt{\frac{ta}{\pi}}\left(1+2\sqrt{\pi}\sum_{n=1}^{\infty}\beta^{n}\,\mathrm{ierfc}\left(\frac{nl}{\sqrt{at}}\right)\right)+\tau \tag{5}$$

and the sensitivity coefficients β_a, β_λ, and β_τ can be calculated using Eq. (4). Figure 3 shows the temperature function and the sensitivity coefficients β_λ and β_a as a function of time. The third coefficient attains a constant value $\beta_\tau = \tau$. Since the sensitivity coefficients are functions of one variable t, the linear dependence can be investigated using Wronsky's determinant [5] given by the form,

$$W(t)=\begin{vmatrix}\beta_a & \beta_\lambda & \beta_\tau \\ \beta'_a & \beta'_\lambda & \beta'_\tau \\ \beta''_a & \beta''_\lambda & \beta''_\tau\end{vmatrix}. \tag{6}$$

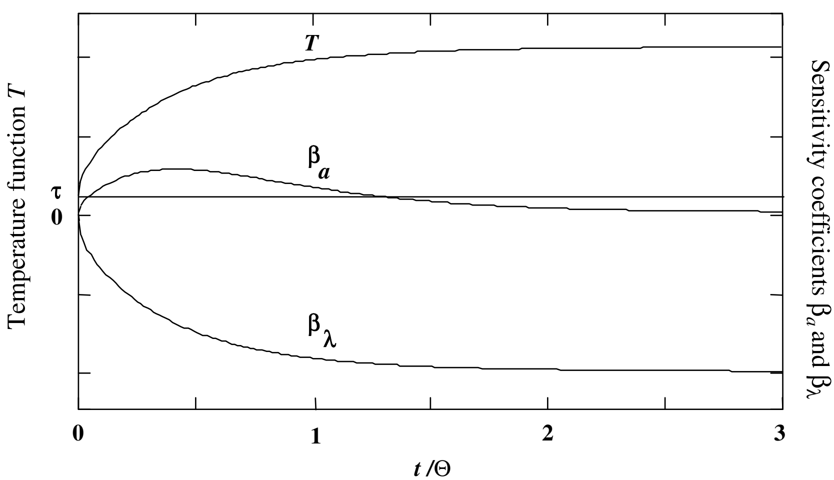

Fig. 3. Temperature function and sensitivity coefficients β_λ and β_a versus nondimensional time scale t/Θ.

The sensitivity coefficients are linearly dependent when the determinant, Eq. (6), is equal to zero. If the functions are represented by equi-spaced time series [6], the derivations can be estimated by the relation,

$$f_i' = \frac{f_{i+1} - f_i}{\Delta t}, \tag{7}$$

where Δt is the time interval between samples. Then the determinant W takes on a very simple form,

$$W_i = c \cdot \begin{vmatrix} \beta_{ai} & \beta_{\lambda i} & 1 \\ \beta_{ai+1} & \beta_{\lambda i+1} & 1 \\ \beta_{ai+2} & \beta_{\lambda i+2} & 1 \end{vmatrix}, \tag{8}$$

where c is a constant. Equation (8) in the form of a time series determines the time interval, where the sensitivity coefficients are not linearly dependent. As seen in Fig. 4, the function W acquires nonzero values in the interval $(0.07\Theta, \Theta)$.

4. DIFFERENCE ANALYSIS

In the previous section we used sensitivity coefficient analysis to determine the time window in which the fitting procedure should be

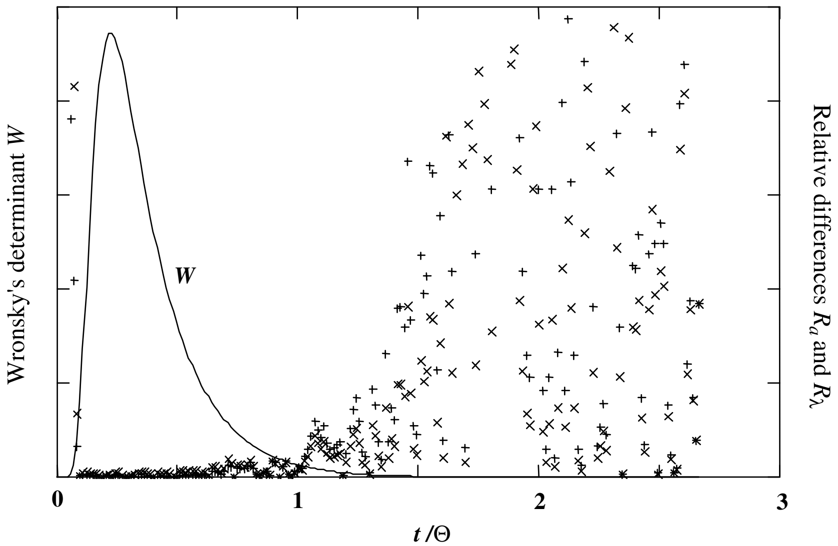

Fig. 4. Values of determinant W and relative differences of the parameters a (x) and λ (+) versus nondimensional time scale t/Θ.

applied to get reliable values of thermophysical parameters a and λ. The difference analysis [7] is another method for the time interval determination. It is based on fitting the theoretical temperature function to the points in the time interval $(t_B, t_B + t_S)$, where t_B and t_S designate the beginning and the size of the interval, respectively (Fig. 2). If t_B is successively increased while t_S is kept constant, a series of parameter values is obtained. If the time interval $(t_B, t_B + t_S)$ is not suitable for the estimation of the parameters a and λ the results of fitting are unreliable and the plot shows considerable scatter.

In order to verify the theory described in the preceding section, we decided to construct a mathematical model of the experiment. In the first stage the points were computed using Eq. (5). Simulating the measurement on polymethylmetacrylate (PMMA), the following values were used: $l = 0.003\,\mathrm{m}$, $q = 1000\,\mathrm{W} \cdot \mathrm{m}^{-2}$, $\lambda = 0.19\,\mathrm{W} \cdot \mathrm{m}^{-1} \cdot \mathrm{K}^{-1}$, $a = 0.12 \cdot 10^{-6}\,\mathrm{m}^2 \cdot \mathrm{s}^{-1}$, $\tau = 0.2\,\mathrm{K}$, and $\beta = -0.954$. The sample period was $T = 1\,\mathrm{s}$, and the number of samples $n = 300$. Noise was added by rounding the temperature coordinate of the points to 7 valid numbers. Then the points were processed by difference analysis with the smallest possible time interval. If we have three unknown parameters in Eq. (5), we need at least three points for evaluation. In this situation we solve a system of three equations instead of fitting. Figure 4 shows a plot of the relative differences, which are defined by the formula,

$$R_x = \left| \frac{x - x_0}{x_0} \right|, \tag{9}$$

where x_0 is the value originally used in the model and x is the value calculated using difference analysis. If the time interval is not suitable for estimation of parameters a and λ, the results are unreliable and relative differences are far from zero.

5. CONCLUSIONS

Figure 4 illustrates the excellent consistency between sensitivity coefficient analysis results, represented by the function W, and difference analysis, represented by relative differences of both parameters a and λ. In the interval $(0.07\Theta, \Theta)$ determinant W acquires nonzero values, so that the sensitivity coefficients are not linearly dependent, the fitting procedure works properly, and computed values are nearly the same as values originally used in the model. Hence, in this time interval, the relative differences of the parameters a and λ, calculated using Eq. (9), are expected to be zero. The lower are the values of determinant W, the higher is the scatter of computed values of thermophysical parmeters a and λ.

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